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Coherent Synchro-Betatron Resonance

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Abstract

Coherent synchro-betatron resonances can present a serious limit for low-energy synchrotrons with strong space charge. Here, an excitation of a dipole transverse mode is considered at resonance condition.

1 Introduction

Rapid cycling sinchrotrons normally deal with so intense beams, that the space charge tune shift can reach or even exceed 0.5. Therefore, to prevent integer resonances, the machine tunes have to be set just a little below of integer numbers. Due to high acceleration rate, these machines have rather high value of the synchrotron tune at injection energy. Small (non-relativistic) injection energy results in the incoherent tune shifts (mainly due to beam space charge) to be smaller the coherent (due to image charges). All these circumstances can lead to an excitation of low-order coherent synchro-betatron resonance (CSBR). Due to the high space charge tune shift, the coherent and incoherent spectra are effectively separated; thus, there is no Landau damping, otherwise stabilizing the resonant mode. CSBR is in a way paradoxical phenomenon: a coherent mode is excited by external perturbations. Growth of this mode does not depend on the beam intensity, as soon as the incoherent spectrum is separated from the coherent line by the space charge, which easily happens at relatively low beam current.

Some indications show that CSBR is an important phenomenon for the Fermilab Booster. This paper does not discuss observations though, leaving that for future reports, and being limited to the very concept of CSBR.

2 Dipole Resonances

Condition of dipole CSBR is $\nu_b + l\nu_s = n$, with $\nu_b = \omega_b/\omega_0$, $\nu_s = \omega_s/\omega_0$ as the coherent betatron and synchrotron tunes, and l, n as integer numbers. When this condition is satisfied, any dipole perturbation generally drives the resonance mode. To understand main features of the phenomenon, a longitudinal distribution is taken here as the air-bag one: all the particles have the same synchrotron amplitude r_0, and they are homogeneously distributed over the synchrotron phases ϕ; the longitudinal offset is $z = r_0 \cos\phi$. Following Ref. [1], the Vlasov equation for the distribution function ψ can be written as

$$\frac{\partial \psi}{\partial s} + \frac{\omega_b}{v_0}\frac{\partial \psi}{\partial \theta} + \frac{\omega_s}{v_0}\frac{\partial \psi}{\partial \phi} + \widetilde{x}'\frac{\partial \psi}{\partial x} + \widetilde{p_x}'\frac{\partial \psi}{\partial p_x} = 0 \ . \tag{1}$$

Here $s = v_0 t$ is time in conventional units of length, v_0 is a longitudinal velocity, $x = q\cos\theta$ and $p_x = -(q/\beta_x)\sin\theta$ are the betatron coordinate and momentum (angle), $\widetilde{x}' = d\widetilde{x}/ds$ and $\widetilde{p_x}' = d\widetilde{p_x}/ds$ are their perturbations. It can be assumed here, that the perturbation kicks are localised at single point $s = s_k$; in case of many points the final result can be obtained by summation over them. Such localized perturbations are presented as

$$\begin{aligned} \widetilde{x}' &= \Delta x \delta_P(s - s_k) \ ; \\ \widetilde{p_x}' &= \Delta p_x \delta_P(s - s_k) \ , \end{aligned} \tag{2}$$

where

$$\delta_P(s) = C^{-1}\sum_m \exp(-ims/R) \tag{3}$$

is the periodical delta-function, $C = 2\pi R$ is the ring circumference.

For example, the perturbations may be caused by acceleration kicks in RF cavities, if there is some dispersion D or its derivative D' there. In this case,

$$\begin{aligned} \Delta x &= -D\sin(kz)\ \Delta p_{\max}/p_0 \\ \Delta p_x &= -(D' + \frac{\alpha_x}{\beta_x}D)\sin(kz)\ \Delta p_{\max}/p_0 \ , \end{aligned} \tag{4}$$

where k is the RF wave-number, α_x and β_x are the local Twiss parameters, $\Delta p_{\max}$ is an amplitude of the RF kick of the longitudinal momentum, and p_0 is the longitudinal momentum itself. Other possibilities for that sort of perturbations include a mismatch between the dipole fields and momentum at acceleration (independent on the longitudinal position z), and dipole fields from image charges and currents (proportional to the local linear density).

According to the conventional perturbation approach, a solution of the Vlasov equation (1) is presented as a sum of a steady state distribution and a perturbation: $\psi = \psi_0 + \widetilde{\psi}$. For the air-bag (hollow beam) distribution,

$$\begin{aligned} \psi_0 &= f_0(q)\delta(r - r_0) \ ; \\ \widetilde{\psi} &= A(s)\sqrt{\beta_x} f_0'(q)\delta(r - r_0)\exp(i\theta + il\phi + i\chi z/r_0 - i\Omega_l s/c) \ . \end{aligned} \tag{5}$$

Here $\Omega_l = \omega_b + l\omega_s = n\omega_0$ is a frequency of the considered resonance mode and $A(s)$ is its slowly growing amplitude, $f_0'(q) = df_0/dq$, and $\chi = \xi r_0/(R\eta)$ is the so-called head-tail phase, with the chromaticity $\xi = d\nu_b/d(\Delta p/p_0)$ and the slippage factor η. Substituting Eqs. (5) in the Vlasov equation (1), neglecting the second-order terms $\propto \widetilde{x}'\widetilde{\psi}$, $\widetilde{p_x}'\widetilde{\psi}$ and leaving only a resonant contribution $m = n$ in the periodical delta-function expansion (3), a time derivative for the mode amplitude is obtained:

$$C\frac{dA}{ds} = \left(\Delta p_x\sqrt{\beta_x}\sin\theta - \frac{\Delta x}{\sqrt{\beta_x}}\cos\theta\right)\exp(-i\theta - il\phi - i\chi\cos\phi + ins_k/R) \ . \quad (6)$$

Averaging over the betatron phases, $e^{-i\theta}\sin\theta \rightarrow 1/(2i)$, $e^{-i\theta}\cos\theta \rightarrow 1/2$, averaging over the synchrotron phases, $\langle ... \rangle_\phi$, and summation over all the perturbations along the ring, $\sum_k$, leads to

$$C\frac{dA}{ds} = \sum_k \frac{\exp(ins_k/R)}{2}\left\langle \left(-i\Delta p_x\sqrt{\beta_x} - \frac{\Delta x}{\sqrt{\beta_x}}\right)\exp(-il\phi - i\chi\cos\phi) \right\rangle_\phi \ . \quad (7)$$

This equation gives an increase per turn of the amplitude of the synchro-betatron dipole mode l in case of the integer resonance $\Omega_l = n\omega_0$. In principle, this phenomenon is equivalent to excitation of a linear oscillator by an external resonance force; the amplitude grows linearly in this case.

In case the resonance is near, but not exact, $\Delta\Omega \equiv \Omega_l - n\omega_0 \neq 0$, the right hand side of Eq. (7) has to be multiplied by an oscillating factor $\exp(i\Delta\Omega s/c)$. The Landau damping can be included adding $-\Lambda_L A$ in the right hand side.

In principle, several ways can be foreseen to suppress CSBR:

- To move the machine tune further from the integer, or put it in between the neighbour CSBRs;
- To increase the chromaticity, reducing the driving force in Eq. (7) and possibly introducing the Landau damping;
- In the opposite, to set the chromaticity so low that $\chi \approx 0$, so that the driving force for either even or odd modes will be zeroed, and to pick such a tune, that the resonance mode is with a zero force;
- To cross CSBR faster;
- To apply a 3rd harmonic in the RF, introducing more Landau damping.

Similarly, CSBR of any betatron order, $m\omega_b + l\omega_s = n\omega_0$, $m \geq 2$, can be treated; however, it is not clear for the authors if the higher order CSBR can be ever significant. Regarding the considered dipole CSBR, we estimated its possible influence for the Fermilab Booster at injection energy, and found it can be pretty strong, leading to the offset growth $\simeq 30 - 60$ turns/mm. Some experimental details have been presented in Ref. [2]; their consideration from a point of view of CSBR is going to be a subject of a separate paper.

3 Conclusion

Analytical consideration of the coherent synchro-betatron resonance is presented for a simplified air-bag distribution. Essentially, the phenomenon results in a linear growth of the resonant coherent mode. This growth is driven by external field, so it is not sensitive to the beam intensity, as soon as Landau damping is switched off by the space charge. The phenomenon tends to be important for low-energy synchrotrons.

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References

[1] A. W. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators", New York: Wiley Inc., 1993.

[2] V. Lebedev, A. Burov, W. Pellico and X. Yang, FERMILAB-CONF-06-205-AD, Jun 2006.